

Countable Sets

①

Equivalent sets:- Let A and B are two nonempty sets and if it is possible to define a one-one onto mapping from A to B or B to A , then A is said to be equivalent to B or B is equivalent to A . we denote it as $A \sim B$, or $B \sim A$. ' \sim ' is called wiggly.

Denumerable set An infinite set A is said to be denumerable if \exists a mapping $f: \mathbb{N} \rightarrow A$ s.t. f is one-one onto.
or if A is equivalent to \mathbb{N} (set of natural numbers, otherwise called non-denumerable).

Countable set: A finite or denumerable set is called countable set.

let
Ex 1) $f: \mathbb{N} \rightarrow \mathbb{N}$ s.t. $f(n) = n \quad \forall n \in \mathbb{N}$
then $f(n)$ is one-one onto

$$\Rightarrow \mathbb{N} \sim \mathbb{N}$$

$\Rightarrow \mathbb{N}$ is denumerable $\Rightarrow \mathbb{N}$ is countable.

Ex 2 Set $\{1, 2, 3, 4\}$ is finite \Rightarrow countable.

Ex 3. $A = \{2^n \mid n \in \mathbb{N}\}$ is countable

Because $\exists f: \mathbb{N} \rightarrow A$ s.t. $f(n) = 2^n$ is one-one onto $\Rightarrow A$ is countable.

Countable sets: \mathbb{R}, \mathbb{C} .

Finite, Denumerable. \rightarrow countable sets.

But \mathbb{R} & \mathbb{C} ~~tends~~ ^{are} uncountable sets.

Th. 1. Any denumerable set can be put into a one-to-one correspondence with a proper subset of itself.

Proof: — let A be a denumerable set.

$\Rightarrow A \sim \mathbb{N}$ or $\mathbb{N} \sim A$.

\Rightarrow All the elements of A can be put as one-to-one correspondence with \mathbb{N} .

$\Rightarrow A = \{a_1, a_2, a_3, \dots\}$. — (i)

let a subset B of A like this

$B = A - \{a_1\} = \{a_2, a_3, a_4, \dots\}$

Now, we define a mapping f , from B to A

$f: B \rightarrow A$ such that.

$f(a_i) = a_{i+1} \quad \forall i \in \mathbb{N}$.

This is one-one-onto.

\therefore If $\bar{a}_i \neq \bar{a}_j$

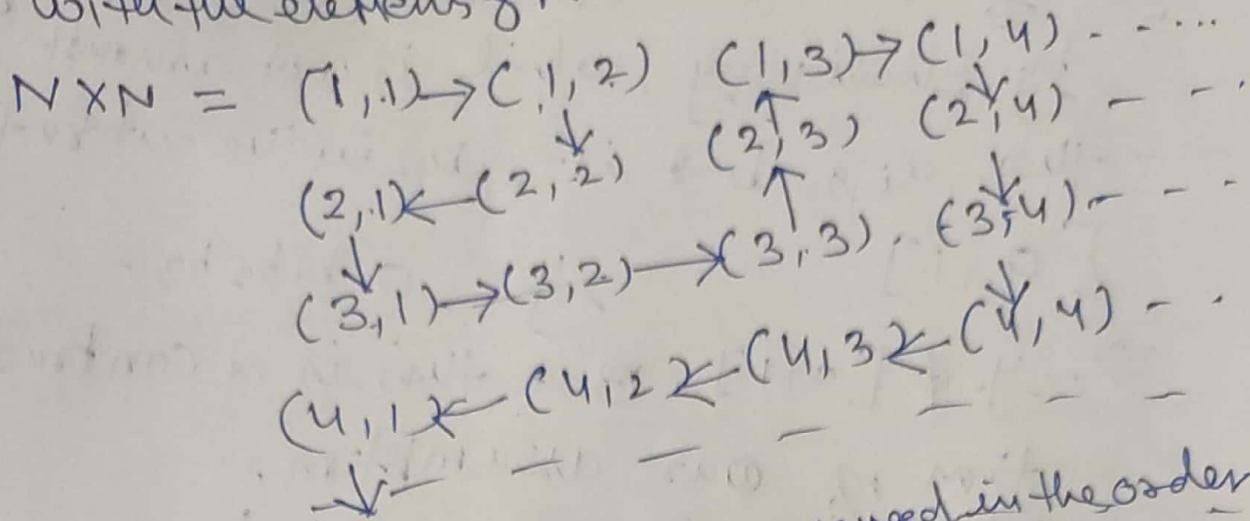
$\Rightarrow f(\bar{a}_i) \neq f(\bar{a}_j)$

$\underline{a_{i+1}} \neq \underline{a_{j+1}}$

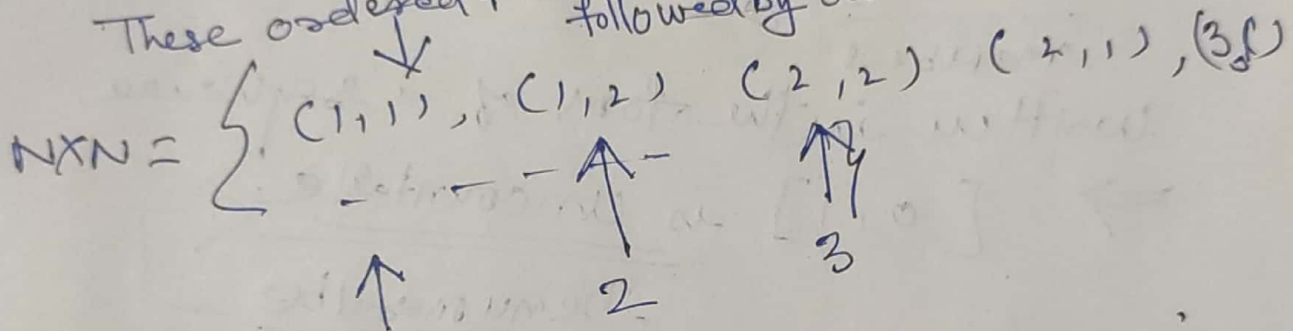
$\Rightarrow \underline{B \sim A} \Rightarrow \underline{A \sim B}$. H.P.

Th 3. The set $\mathbb{N} \times \mathbb{N}$ is countable.

$\mathbb{N} \times \mathbb{N} =$ set of ^{all} ordered pairs, formed with the elements of \mathbb{N} .

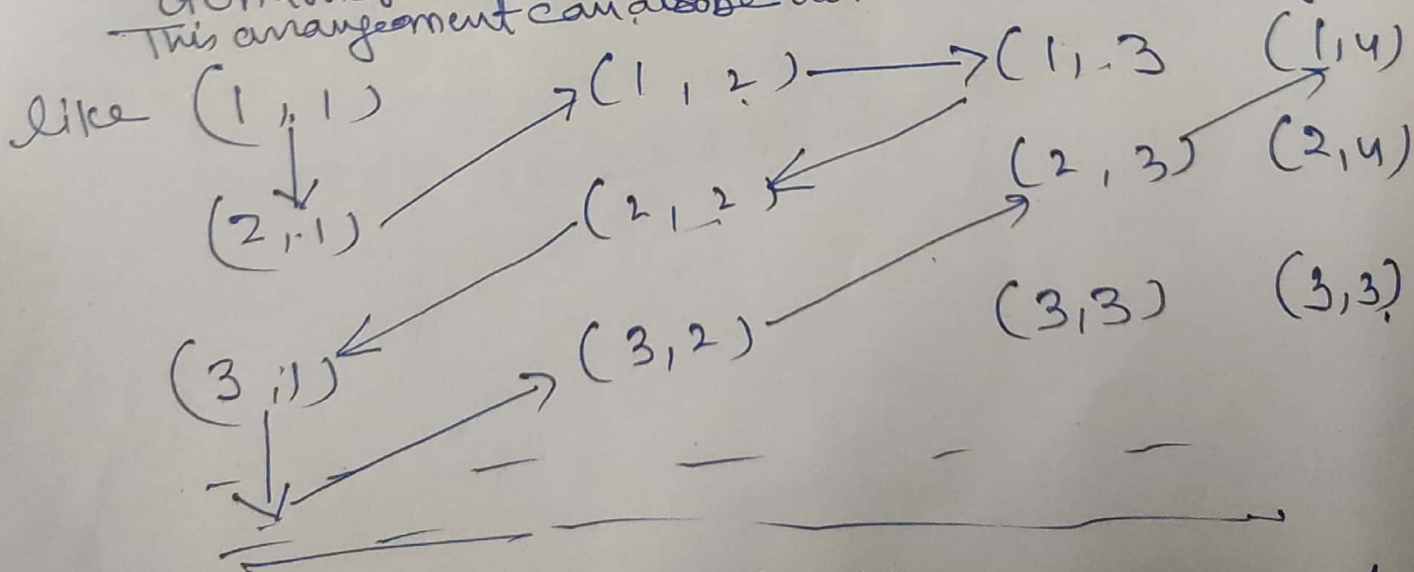


These ordered pairs can be arranged in the order followed by arrows, i.e.



$\mathbb{N} \times \mathbb{N} \sim \mathbb{N} \Rightarrow \mathbb{N} \sim \mathbb{N} \times \mathbb{N} \Rightarrow \mathbb{N} \times \mathbb{N}$ is denumerable \Rightarrow Countable

This arrangement can also be done in another way also.



In this way these ordered pairs can be arranged in the form of sequence $\Rightarrow \mathbb{N} \times \mathbb{N}$ is Countable.