Countable Sets

Equivalent sets:- Let A and B are two nonempty sets and if it is possible to define a one-one onto mapping from A to B or B to A, then A is said to be equivalent to B or B is equivalent to A. we denote it as $A \sim B$, or $B \sim A$.

'$\sim$' is called wiggle.

Denumerable set An infinite set A is said to be denumerable if $\exists$ a mapping $f: N \to A$ s.t. f is one-one onto.

or if A is equivalent to N (set of natural numbers). otherwise called non-denumerable.

Countable set: A finite or denumerable set is called Countable set.

Ex (1) Let $f: N \to N$ st. $f(x) = x \ \forall x \in N$

then $f(x)$ is one-one onto

$\Rightarrow N \sim N$

$\Rightarrow$ N is denumerable $\Rightarrow$ N is Countable.

Ex 2 Set $\{1, 2, 3\}$ is finite $\Rightarrow$ Countable.

Ex 3. $A = \{2^n \mid n \in N\}$ is countable

Because $\exists f: N \to A$. st. $f(n) = 2^n$ is one-one onto $\Rightarrow$ A is Countable.

Countable sets:

$\underline{R}, \underline{C}.$

Finite, Denumerable. $\longrightarrow$ countable sets.

But R & C ~~tends~~ are uncountable sets.

Th. 1. Any denumerable set can be put into a one-to-one correspondence with a proper subset of itself.

Proof:— Let A be a denumerable set.

$\Rightarrow A \sim N.$ or $N \sim A.$

$\Rightarrow$ All the elements of A can be put as one-to-one correspondence with N.

$\Rightarrow A = \{a_1, a_2, a_3, \dots\}$ —— (i)

Let a subset B of A like this

$B = A - \{a_1\} = \{a_2, a_3, a_4, \dots\}$

Now, we define a mapping f, from B to A

$f: B \to A.$ such that.

$f(a_i) = a_{i+1} \quad \forall i \in N.$

This is one-one-onto.

$\because$ If $a_i \neq a_j$

$\Rightarrow f(a_i) \neq f(a_j)$

$\underline{a_{i+1}} \neq \underline{a_{j+1}}$

$\Rightarrow \underline{B \sim A} \Rightarrow \underline{A \sim B}.$ H.P.

Th 3. The set $N \times N$ is countable.

$N \times N$ = set of all ordered pairs formed with the elements of N.

$$
\begin{array}{lllll}
N \times N = & (1,1) \to (1,2) & (1,3) \to (1,4) & \cdots \\
 & (2,1) \leftarrow (2,2) & (2,3) & (2,4) & \cdots \\
 & (3,1) \to (3,2) \to & (3,3) & (3,4) & \cdots \\
 & (4,1) \leftarrow (4,2) \leftarrow & (4,3) \leftarrow & (4,4) & \cdots \\
 & \downarrow & & &
\end{array}
$$

These ordered pairs can be arranged in the order followed by arrows. i.e.

$$N \times N = \{ (1,1), (1,2), (2,2), (2,1), (3,1) \ldots \}$$

$$\uparrow \qquad\quad \uparrow \qquad\quad \uparrow \qquad \uparrow$$

$$1 \qquad\quad 2 \qquad\quad 3 \qquad 4$$

$N \times N \sim N \Rightarrow N \sim N \times N \Rightarrow N \times N$ is denumerable $\Rightarrow$ Countable.

This arrangement can also be done in an another way also like

$$
\begin{array}{llll}
(1,1) & (1,2) \to & (1,3) & (1,4) \\
(2,1) & (2,2) & (2,3) & (2,4) \\
(3,1) & (3,2) & (3,3) & (3,3) \\
\downarrow & & &
\end{array}
$$

In this way these ordered pair can be arranged in the form of sequence $\Rightarrow N \times N$ is Countable.